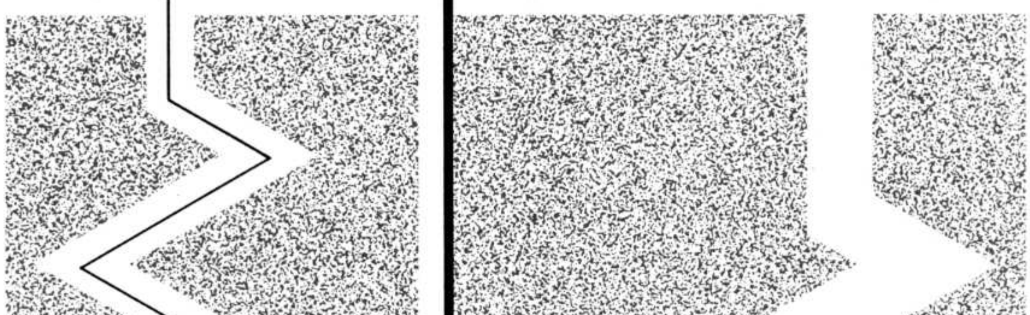
NOISE IN PRECISION FILM RESISTORS

BY P. D. SMITH



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1. GENERAL

Noise in resistors is considered to be made up of two parts: thermal noise, which is caused by random motion of electric charge in the resistor and current noise, which is caused by fluctuations in conductivity of the resistor.

Thermal noise is present in all types of resistors and can only be reduced by lowering the temperature. Current noise is inversely proportional to frequency and may be much larger than thermal noise at low frequencies. However, at high frequencies current noise is negligible compared to thermal noise so that all types of resistors are equally good with respect to high frequency noise. The sketch in Fig. 1 shows this relationship between current noise and thermal noise.

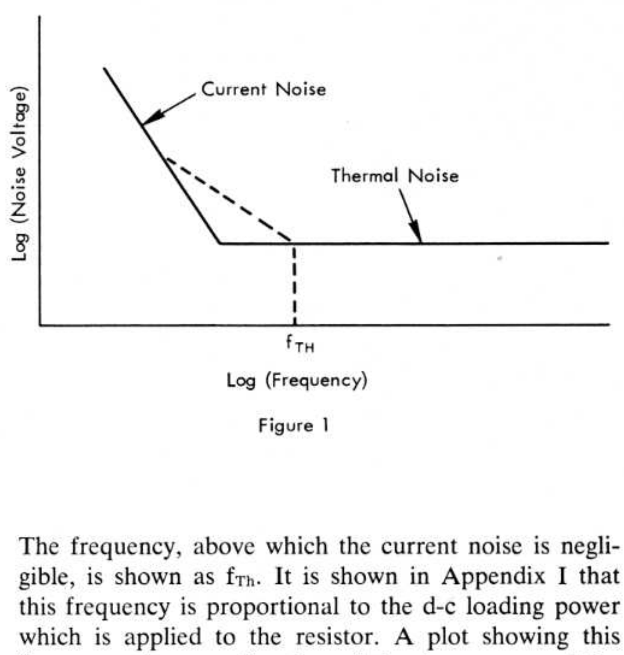

Figure 1

The frequency, above which the current noise is negligible, is shown as f_{Th}. It is shown in Appendix I that this frequency is proportional to the d-c loading power which is applied to the resistor. A plot showing this frequency, f_{Th}, as a function of the d-c power and the Noise Index for the resistor is shown in Fig. 2.

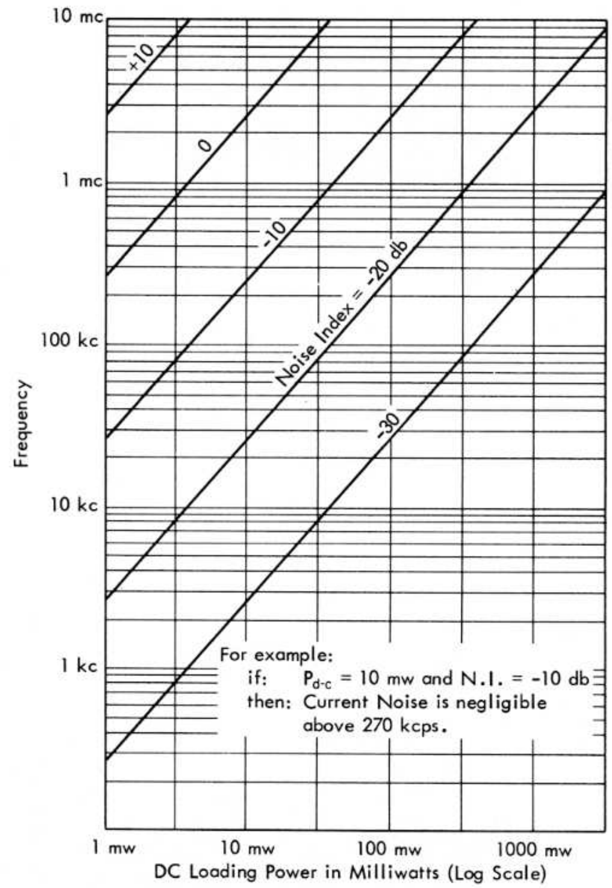

Figure 2

Current Noise is Negligible for Frequencies Above Line at the Given DC Loading Power

As an example of the use of Fig. 2, a resistor which has a Noise Index of – 10 db and a d-c loading power of 10 milliwatts will have a negligible current noise spectral density for frequencies above 270 kilocycles per second.

2. THERMAL NOISE

Thermal noise is "white noise" because all frequencies are present and have the same amplitude. Since thermal noise is caused by random motion of electric charge, the noise voltage may be calculated from thermodynamic considerations (Ref. 1). It has also been experimentally verified that the equation for the mean square value of the thermal noise voltage of a resistor is:

1. $\overline{e_{th}^2} = 4kT\ \Delta fR$, where e_{th}^2 is the mean sq. voltage in the band Δf

   k is Boltzmann's constant, 1.38×10^{-23} Joules/°K

   T is the temperature of the resistor in °K (0°K = – 273°C)

   Δf is the effective bandwidth in cps

   R is the resistance in ohms.

At room temperature of 25°C, the mean square thermal noise is

2. $\overline{e_{th}^2} = 1.645 \times 10^{-20} \Delta f R$.

It should be noted that this is the open circuit voltage. The noise voltage at the terminals will be determined by the other components in the particular circuit.

3. CURRENT NOISE

Current noise, which is present in a resistor only when the resistor is carrying a current, is attributed to fluctuations in the resistance value of the resistor. This could be caused by fluctuations in the conductivity of the resistive material, as is apparently the case in transistors, or by a variation in contact resistance as in the carbon microphone. Current noise is negligible in wire-wound resistors.

The mean square value of current noise has a frequency spectrum which is nearly proportional to l/f. Therefore, current noise predominates at low frequencies but is negligible compared to thermal noise at high frequencies as was shown in Fig. 1. The rms value of the current noise is proportional to the d-c voltage across the resistor.

The equation for the mean square value of the current noise is

$$3.\ \frac{\overline{e_c^2}}{f} = K\frac{E^2}{f} = K\frac{I^2R^2}{f}$$

where K is a constant for the particular resistor type

$\overline{e_c^2}$ is the mean square value of the current noise

I is the d-c current in the resistor

E is the d-c voltage across the resistor

f is the frequency in cps

R is the resistance in ohms.

The l/f current noise power spectrum has been verified to as low as 2.5×10^{-4} cps according to some authors (Ref. 6). It obviously cannot go to d-c since this would require an infinitely large value of mean squared noise.

4. TOTAL RESISTANCE NOISE

Resistor noise may be expressed in terms of the noise power spectral density (NPSD) which is the noise power produced in a 1-ohm resistor in the frequency band Δf. The factor 10^{12} is thrown in so that the units are microvolts squared per cps.

$$4.\ \text{NSPD} = \frac{\overline{e_n^2}}{\Delta f} \times 10^{12}\ \frac{(\mu v)^2}{cps}$$

The total resistance noise power spectral density is the sum of the thermal noise power from equation 1 and the current noise power from equation 3.

$$5.\ \text{NPSD} = \frac{\overline{e_n^2}}{\Delta f} \times 10^{12} = \left(4KTR + K\frac{E^2}{f}\right)\frac{(\mu v)^2}{cps} \times 10^{12}$$

The NPSD may be converted into decibels relative to 1 v^2/cps as follows.

$$6.\ (\text{NPSD})_{db} = 10 \text{ Log}\frac{\overline{e_n^2}}{\Delta f} = 10 \text{ Log (NPSD)} \times 10^{-12}$$

$$= 10 \text{ Log}\left(4KTR + K\frac{E^2}{f}\right)$$

The NPSD in decibels is plotted as a function of log f in Fig. 3, where the current noise term and the thermal noise term are plotted separately. The slope of the low-frequency current noise is − 3db per octave (− 10db per decade).

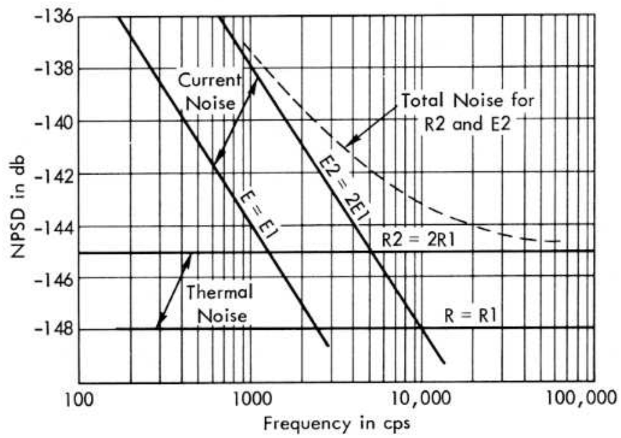

Figure 3

Dependence of Noise on Resistance and Voltage

The thermal noise level increases 3 db for each doubling of the resistance value, R, and the current noise increases 6 db for each doubling of the d-c voltage, E.

5. UNITS USED TO EXPRESS CURRENT NOISE

Several units have been proposed to express the current noise of resistors. The unit of microvolts per volt as designated in the JAN-R-11 specification has been used for some time but the measurement depends on the particular test set and the unit is not easily used to predict the noise which a resistor will produce in a particular amplifier. The conversion gain has been proposed by George T. Conrad, Jr. of the Bureau of Standards and appears to give consistent noise readings on different test setups and may also be used to predict the noise from one particular amplifier. The Noise Index is a more recently proposed unit which uses the same measurements setup as the conversion gain but is more readily interpreted to predict the resistor noise in a particular circuit.

We have chosen the Noise Index (N.I.) as the most useful measure of current noise in a resistor.

The conversion equation to obtain the Conversion Gain, G_c, from the Noise Index is derived in Appendix II and shown as equation 7 below.

7. $(G_c)_{db} = (N.I.)_{db} - 159.6$ db.

The relationship between the old JAN-R-11 specification of microvolts per volt and the Noise Index is shown in Fig. 4 which was taken from Conrad (ref. 5). The relationship depends on the resistance value which is included on Fig. 4 and also somewhat on the particular JAN-R-11 type of test setup which cannot be allowed for on the graph.

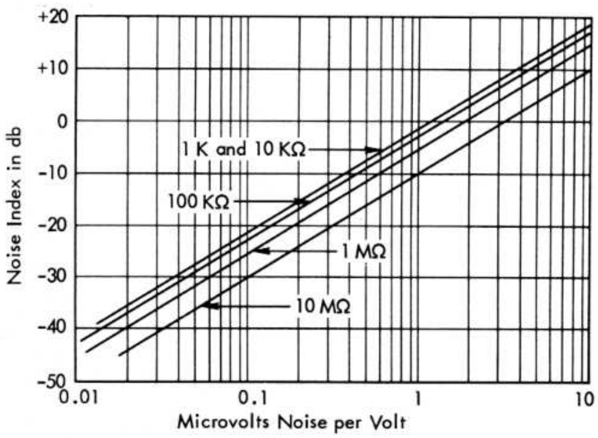

Figure 4

Relationship Between Noise Index and Microvolts per Volt

6. NOISE INDEX (N.I.)

The Noise Index is defined as the number of microvolts of current noise per volt of d-c voltage in one decade of bandwidth. Since the current noise power is given by $\frac{\overline{e_c^2}}{\Delta f} = \frac{KE^2}{f}$, the total current noise in the frequency band extending from f_1, to f_2 is

8. $$\overline{E_c^2} = \int_{f_1}^{f_2} \frac{KE^2}{f}\, df = KE^2 \, Log_e \, f_2/f_1$$

If $f_2 = 10f_1$ which is a decade band, then

9. $\overline{E_c^2} = KE^2 \, 2.3$

and the Noise Index is then

10. $$N.I. = \sqrt{\frac{\overline{E_c^2}}{E^2}} \times 10^6 = \sqrt{2.3K} \times 10^6 \frac{\mu v}{v} \Big/ \text{decade}$$

which is a direct measure of K or the current noise from the resistor. The Noise Index is not a function of frequency although it is usually measured at 1000 cps.

The Noise Index in decibels is given by

11. $$(N.I.)_{db} = 10 \, Log_{10} \frac{E_c^2}{E^2} \times 10^{12}$$

12. $$= 20 \, Log_{10}(E_c \times 10^6) - 20 \, Log_{10}E$$

= (Noise Voltage in db above luv/decade) – (d-c voltage in db)

7. CALCULATION OF RESISTOR NOISE IN A PARTICULAR CIRCUIT

The resistor noise produced in a particular circuit may be calculated from known values of Noise Index, resistance value, d-c loading power, and the frequency band of the circuit.

For example, consider an amplifier with a 100-kilohm resistor with 100 volts across it in the plate circuit of the input pentode. Assume also that the Noise Index of the resistor is – 10 db and the frequency band of the amplifier extends from 20 to 10,000 cps.

Then the Noise Index in microvolts per volt per decade is determined from

$-10 \text{ db} = 20 \, Log_{10} \, (N.I.)$ or

$N.I. = 0.316 \; \mu v/v/\text{decade}$.

The frequency band ratio for this amplifier is

$$\frac{10{,}000}{20} = 500$$

and this may be used with Fig. 5 to determine the Bandwidth Factor (B.F.) for this amplifier. (A one-decade bandwidth ratio (10) would have a bandwidth factor of 1. The equation for Fig. 5 is derived in Appendix III.)

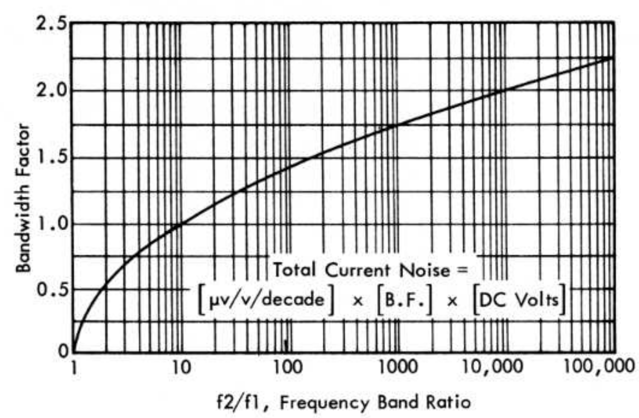

Figure 5

Bandwidth Factor (B.F.) for 1/f Noise

Therefore, from Fig. 5 the bandwidth factor is 1.65 and the total current noise in the frequency band is

$0.316 \times 1.65 \times 100 = 52.2 \; \mu v$.

From equation 2 the thermal noise from the resistor at room temperature is

$$e_{th} = \sqrt{4kTR\,\Delta f}$$
$$= \sqrt{1.645 \times 10^{-20} \times 10^5 \times (10{,}000\text{-}20)}$$
$$= 4.05\ \mu v$$

Since the noise sources are independent, the total resistor noise voltage in the frequency band is equal to the square root of the rms of the squares of the noise voltages.

$$E_T = \sqrt{(4.05)^2 + (52.2)^2}$$
$$= 54\ \mu v \text{ which is almost entirely current noise.}$$

It should be remembered, of course, that this is the Thevinen's Equivalent or "open circuit" voltage in the resistor so that the actual voltage at the terminals will depend on the other impedances in the circuit.

8. APPLICATIONS FOR LOW NOISE CARBON FILM RESISTORS

The primary use for low noise resistors is in the input stage of an amplifier. Only thermal noise is present in a resistor which is not carrying a current and all resistor types should work equally well with respect to noise. For a resistor which is carrying a current, such as a plate collector or base resistor, current noise may be present in addition to thermal noise.

For amplifiers that operate in a frequency range above the useful range of a wire-wound resistor (above about 50 Kcps), the deposited carbon film resistor is applicable as a low noise resistor. As seen from Fig. 1, if the d-c loading power is 5 milliwatts or less and the Noise Index for the resistor is -10 db, then the current noise is negligible for frequencies above 50 Kcps.

If the d-c loading power is kept reasonably low, the deposited carbon resistor will have no more noise than a wire-wound resistor. This maximum loading power for a given frequency may be determined from Fig. 1. Values of Noise Index for the different types of TI resistors are shown in Fig. 6.

9. PEAK INSTANTANEOUS NOISE

Since noise is a fluctuating quantity, the peak value of the noise is more indicative of performance than the rms noise for some circuits (such as an oscilloscope). The peak-to-rms ratio, which is exceeded a given percent of the time, is shown in Table 1 below (from Ref. 7).

Table 1

Percent of Time Peak is Exceeded	Peak/RMS
10.00	1.645
1.00	2.576
0.10	3.291
.01	3.890
.001	4.418
.0001	4.892

From Table 1, it is seen that a high peak value is likely to be found and that some engineering judgment is needed to determine a reasonable peak-to-rms ratio for a particular application.

10. AVERAGE NOISE INDEX OF TI RESISTORS

The average Noise Index of TI precision film resistors is shown in Figure 6. In general the higher power rated resistors (1w, 2w) have less noise than the 1/8, 1/4, or 1/2 watt resistors although this is not shown in Figure 6. The standard deviation of the Noise Index for any group of ten identical resistors is about 2.5 db, however, the average Noise Index for the group may be as much as 10 db different from the lines shown.

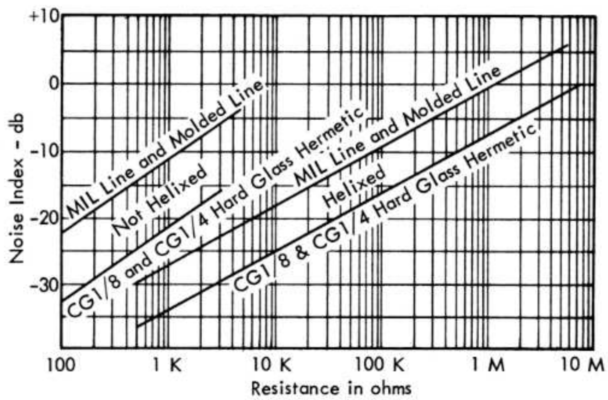

Figure 6

Average Noise Index for Texas Instruments Precision Film Resistors

APPENDIX I

RELATIONSHIP BETWEEN DC LOADING POWER AND FREQUENCY ABOVE WHICH CURRENT NOISE IS SMALL COMPARED TO THERMAL NOISE

Since the current noise power spectral density is inversely proportional to the frequency and the thermal noise power spectral density is constant, a frequency may be found above which the current noise is small compared to the thermal noise. If this frequency is chosen such that the current noise power is 10% of the thermal noise power, then

$$\frac{\overline{e_c^2}}{\Delta f} = 0.1\,\frac{\overline{e_{th}^2}}{\Delta f}$$

$$K\frac{E^2}{f} = 0.1 \times (4kT)R.$$

Then solving this for the d-c power yields

$$P = \frac{E^2}{R} = \frac{0.1\ (4kT)}{k}\, f_{th}.$$

The factor K may be eliminated from

$$(\text{N.I.})_{db} = 10 \text{ Log}_{10} \frac{\overline{E_c}^2}{E^2} = 10 \text{ Log}_{10}\, 2.3\text{K} \times 10^{12}$$

from which $K = \frac{1}{2.3 \times 10^{12}} 10^{\,NI/10}$

Therefore,

$$P = \frac{0.1\ (4kT)\ 2.3 \times 10^{12}}{10^{NI/10}} f_{th},$$

and since $4kT = 1.645 \times 10^{-20}$ at room temperature,

$$P = \frac{0.369 \times 10^{-8}}{10^{NI/10}} f_{th}$$

if $NI = 0$ db, then

$$P = 0.369 \times 10^{-8}\ f_{Th} \text{ or}$$

$$f_{Th} = 2.71 \times 10^{8}\ P$$

For each -10 db change in the Noise Index, the frequency (f_{Th}) is decreased by a factor of 10. Figure 2 is a plot of this relationship.

APPENDIX II

RELATIONSHIP BETWEEN CONVERSION GAIN AND NOISE INDEX

CONVERSION GAIN

A useful method of describing the current noise from a resistor has been proposed by George T. Conrad, Jr. in Ref. 3.

The current noise is expressed by Conversion Gain, Gc, which is a measure of how efficiently d-c power is converted into noise power.

The Conversion Gain is defined as

13. $$G_c = 10 \text{ Log} \frac{\text{available current noise power}}{\text{DC power}}$$

Since the available current noise power is given by $\frac{\overline{e_c}^2}{4R\Delta f}$ and the d-c loading power is given by $\frac{E_{DC}^2}{R}$, the Conversion Gain is

14. $$G_c = 10 \text{ Log} \frac{\overline{e_c}^2/\Delta f}{4E^2}$$

$$= 10 \text{ Log} \left(\frac{\overline{e_c}^2}{\Delta f} \times 10^{12}\right) - 20 \text{ Log } E - 10 \text{ Log } 4 \times 10^{12}$$

$$= (\text{Noise Voltage in db above} \frac{1\,\text{uv}}{\text{cps}}) - (\text{d-c voltage in db}) - 126 \text{ db}$$

Since the current noise power is proportional to E^2, the Conversion Gain, G_c, is independent of the d-c voltage. Conrad has also shown (in Ref. 2) that the conversion gain in db has a Gaussian distribution so that it may be expressed in terms of an average value and the standard deviation.

The Conversion Gain must be given at a particular frequency (usually 1000 cps) because the current noise is function of frequency.

RELATIONSHIP BETWEEN NOISE UNITS

The Noise Index in decibels is given in equations 10 and 11 as

$$(\text{N.I.})_{db} = 10 \text{ Log} \frac{(\overline{E_c}^2}{E^2} \times 10^{12}) = 10 \text{ Log } (2.3\text{K} \times 10^{12})$$

and the Conversion Gain in decibels is obtained from equation 3 and 14 as

$$G_c = 10 \text{ Log} \frac{\overline{e_c}^2/\Delta f}{4E^2} = 10 \text{ Log} \frac{K}{4f}.$$

These equations may be combined to give

$$G_c = (NI)_{db} - 10 \text{ Log } (2.3\text{K} \times 10^{12}) + 10 \text{ Log} \frac{K}{4f}$$

$$= (NI)_{db} - 10 \text{ Log } (2.3 \times 4 \times f \times 10^{12}).$$

Since G_c is measured at 1000 cps, $f = 1000$ cps and

$$G_c = (NI)_{db} - 10 \text{ Log } (2.3 \times 4 \times 1000 \times 10^{12})$$

15.

$$G_c = (NI)_{db} - 159.64\text{db}$$

Therefore, the Noise Index is converted to Conversion Gain by subtracting a constant factor of 159.64 decibels.

APPENDIX III

RELATIONSHIP BETWEEN FREQUENCY BAND RATIO AND BANDWIDTH FACTOR

It has been shown in Section 6, equation 8, that the mean square current noise in a frequency band extending from f_1 to f_2 depends on the ratio of f_2/f_1 according to the equation

$$\overline{E_c}^2 = KE^2 \text{ Log}_e\, f_2/f_1.$$

However, the current noise has been measured in terms of the rms noise voltage per decade of band width. From equation 9

$$\overline{E_c}^2{}_{1\ \text{decade}} = KE^2\ 2.3.$$

By combining these two equations and putting in E_{DC}^2 we obtain

$$\overline{E_c^2} = \left(\frac{E_c{}^2{}_{1\ \text{decade}}}{E_{DC}{}^2}\right)\left(\frac{\text{Log}_e\, f_2/f_1}{2.3}\right) E_{DC}{}^2, \text{ or}$$

$$16.\quad E_c = \frac{E_{c\ 1\ \text{decade}}}{E_{DC}} \times \frac{\text{Log}_e\, f_2/f_1}{2.3}\, E_{DC}$$

The quantity in the brackets is the Noise Index and the square root quantity is the Bandwidth Factor (B.F.). Therefore,

$$17.\quad E_c = (\text{N.I.}) \times (\text{B.F.}) \times E_{DC}$$

and the Bandwidth Factor is

$$18.\quad (\text{B.F.}) = \sqrt{\frac{\text{Log}_e\, f_2/f_1}{2.3}}$$

This equation is plotted in Figure 5.

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